

1. Simplify each of the following. Express each answer with positive exponents.

$$a) \left(\frac{a^3}{b^4} \right)^{-2}$$

$$= \frac{a^{-6}}{b^8}$$

$$= \frac{1}{a^6 b^8}$$

$$b) \frac{-12x^{-3}(xy^2)^{-1}}{(2x^3y^{-1})^3}$$

$$= \frac{-12x^{-3}(x^{-1}y^{-2})}{8x^9y^{-3}}$$

$$= \frac{-12x^{-4}y^{-2}}{8x^9y^{-3}}$$

$$= -\frac{3y}{2x^{13}}$$

2. Evaluate without the use of a calculator. Express answers as integers or rational numbers.

$$a) 9^{\frac{3}{2}}$$

$$= \frac{1}{(\sqrt{9})^3} = \frac{1}{27}$$

$$= \frac{1}{3^3}$$

$$b) 64^{\frac{1}{2}} \times 27^{\frac{1}{3}}$$

$$= \sqrt[3]{64} \times \frac{1}{\sqrt[3]{27}}$$

$$= 8 \times \frac{1}{3}$$

$$= \frac{8}{3}$$

3. Solve for the unknown:

$$a) 27^x = \frac{1}{9}$$

$$3^{3x} = 3^{-2}$$

$$3x = -2$$

$$x = -\frac{2}{3}$$

$$b) 2^{x+4} + 2^x = 136$$

$$2^x(2^4 + 1) = 136$$

$$2^x(17) = \frac{136}{17}$$

$$2^x = 8$$

$$x = 3$$

4. Expand and simplify $2(a+1)^2 - (2a-1)(2a+1)$

$$= 2(a^2 + 2a + 1) - (4a^2 + 2a - 2a - 1)$$

$$= 2a^2 + 4a + 2 - 4a^2 + 1$$

$$= -2a^2 + 4a + 3$$

5. Simplify and state any restrictions on the variable.

$$a) \frac{a^2 + 13a + 30}{a^2 - 4a} \times \frac{a - a^2}{a^2 + 2a - 3}$$

$$= \frac{(a+3)(a+10)}{a(a-4)} \times \frac{a(1-a)}{-1(a-1)(a+3)}$$

$$= \frac{-(a+10)}{(a-4)}, a \neq 4, 0, 1, -3$$

6. Simplify: $\frac{d^2 - 2d - 3}{2d^2 + d - 1} \cdot \frac{3d - 1}{2d^2 - 7d + 3}$

$$\frac{(d-3)}{(d-3)} = \frac{\cancel{(d+1)}(d-3)}{(2d-1)\cancel{(d+1)}} \cdot \frac{3d-1}{(2d-1)(d-3)}$$

$$= \frac{d^2 - 6d + 9 - 3d + 1}{(d-3)(2d-1)}$$

$$= \frac{d^2 - 9d + 10}{(d-3)(2d-1)}, d \neq 3, \frac{1}{2}, -1$$

$$b) \frac{3x}{x-3} + \frac{2x}{x+3} \cdot \frac{(x-3)}{(x-3)}$$

$$= \frac{3x^2 + 9x + 2x^2 - 6x}{(x+3)(x-3)}$$

$$= \frac{5x^2 + 3x}{(x+3)(x-3)}, x \neq 3, -3$$

7. Simplify to mixed radical form.

$$a) 2\sqrt{6} \times 3\sqrt{8}$$

$$= 6\sqrt{48}$$

$$= 6\sqrt{16 \times 3}$$

$$= 24\sqrt{3}$$

$$b) \frac{12\sqrt{60}}{3\sqrt{12}}$$

$$= 4\sqrt{5}$$

$$c) \frac{4\sqrt{27} - 6}{6}$$

$$= \frac{4\sqrt{9 \times 3} - 6}{6}$$

$$= \frac{12\sqrt{3} - 6}{6}$$

$$= 2\sqrt{3} - 1$$

8. Solve for x by factoring or using the quadratic formula: Express answer in exact simplified form.

$$a) 6x^2 + 11x - 2 = 0$$

$$x = \frac{-11 \pm \sqrt{(11)^2 - 4(6)(-2)}}{2(6)}$$

$$x = \frac{-11 \pm \sqrt{169}}{12}$$

$$x = \frac{-11 \pm 13}{12}$$

$$x = -2$$

$$x = \frac{1}{6}$$

$$b) 3x^2 = 2x - 4 \rightarrow 3x^2 - 2x + 4 = 0$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(3)(4)}}{2(3)}$$

$$x = \frac{2 \pm \sqrt{-44}}{6}$$

\therefore No real roots

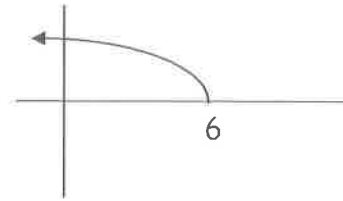
9. State the domain and range of each of the following:

a) $y = 2(x-3)^2 + 4$

$$D \rightarrow \{x \in \mathbb{R}\}$$

$$R \rightarrow \{y \in \mathbb{R} \mid y \geq 4\}$$

b)

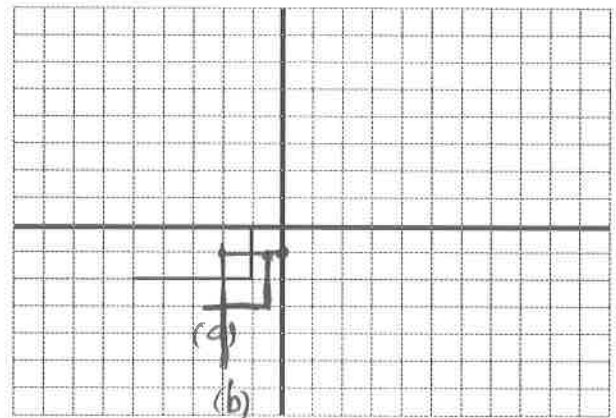


$$D \rightarrow \{x \in \mathbb{R} \mid x \leq 6\}$$

$$R \rightarrow \{y \in \mathbb{R} \mid y \geq 0\}$$

10. Given the graph below of the function $f(x)$, sketch on the same grid (label each clearly):

a) $y = f(2x) - 1$



b) $y = f^{-1}(x)$

11. a) Determine the inverse of the function $f(x) = \sqrt{x-1}$

$$x = \sqrt{y-1}$$

$$x^2 = y - 1$$

$$f^{-1}(x) = x^2 + 1$$

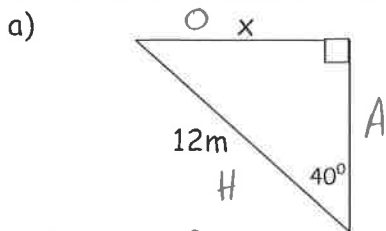
$$x^2 + 1 = y$$

b) What is the domain and range of the inverse?

$$D \rightarrow \{x \in \mathbb{R}\}$$

$$R \rightarrow \{y \in \mathbb{R} \mid y \geq 1\}$$

12. Solve for the unknowns in the diagrams. Express answer to the nearest tenth.

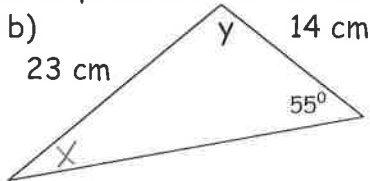


$$\sin \theta = \frac{o}{H}$$

$$\sin 40^\circ = \frac{x}{12}$$

$$12 \times \sin 40^\circ = x$$

$$7.7m = x$$



$$\frac{\sin 55}{23} = \frac{\sin X}{14}$$

$$\sin X = \frac{14 \times \sin 55}{23}$$

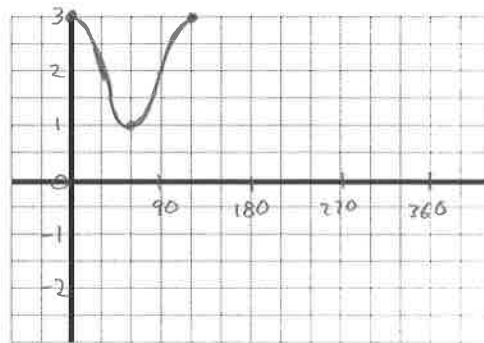
$$\sin X = 0.4986$$

$$X = 30^\circ$$

$$\angle Y = 180 - 55 - 30$$

$$= 95^\circ$$

13. Sketch one cycle (or period) of $y = \cos 3x + 2$, showing all critical points.



14. State the amplitude, period, phase shift and vertical shift for each of the following (where applicable):

a) $y = 2\sin(\theta - 150^\circ)$

b) $y = -\cos \frac{1}{3}(x + 45^\circ) + 1$

2 - Amplitude - 1

360 - Period - 1080 $\left(\frac{360}{\frac{1}{3}}\right)$

Right 150° - Phase Shift - Left 45°

0 - Vertical Shift - Up 1

15. Find any angles that satisfy the following. Give exact answers.

a) $\sin x = 0.5$, $0 \leq x \leq 360^\circ$

$$x = 30^\circ$$

$$x = 180 - 30$$

$$x = 150^\circ$$

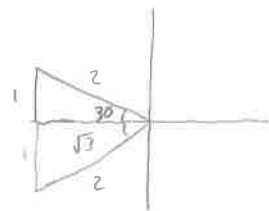
b) $\cos x = -\frac{\sqrt{3}}{2}$, $0^\circ \leq x \leq 360^\circ$

$$x = 180 - 30$$

$$= 150^\circ$$

$$x = 180 + 30$$

$$= 210^\circ$$



16. Find the indicated term:

a) $t_n = 1 - 2n^2$; t_5

$$\begin{aligned} t_5 &= 1 - 2(5)^2 \\ &= 1 - 2(25) \\ &= -49 \end{aligned}$$

b) 12, 19, 26, 33, ... t_{15}

$$\begin{aligned} a &= 12 \\ d &= 7 \end{aligned}$$

$$t_n = 12 + (n-1)7$$

$$\begin{aligned} t_{15} &= 12 + (15-1)7 \\ &= 12 + (14)7 \\ &= 12 + 98 \\ &= 110 \end{aligned}$$

17. Find the indicated sum using an appropriate formula:

a) $S_{12} = 35 + 32 + 29 + \dots$

$$a = 35 \quad d = -3$$

$$\begin{aligned} S_{12} &= \frac{12}{2} (2(35) + (12-1)(-3)) \\ &= 6(70 + (-33)) \\ &= 222 \end{aligned}$$

b) $S_{12} = 36864 - 55296 + 82944 - 124416 + \dots$

$$a = 36864 \quad r = -1.5$$

$$\begin{aligned} S_{12} &= \frac{36864((-1.5)^{12} - 1)}{(-1.5) - 1} \\ &= \end{aligned}$$

18. Suppose that Carolyn wants to deposit enough money today in a savings account so she will have \$1,000,000 in 40 years. If the account pays 7% p.a. compounded semi-annually, how much does she need to deposit?

P.V.	(?) 63 792.85
Payment	—
F.V.	1 000 000
rate	7
Periods	80
Compound	Semi-Annual

19. If Jon deposits \$100 at the end of every three months, how much will he have accumulated after 8.5 years if interest is 6% p.a. compounded quarterly?

P.V	—
P _{mt}	-100
FV	? <u>4393.31</u>
Rate	6
Periods	34 (8.5×4)
Comp.	Quarterly

20. The height above the ground (in metres) of a rocket is given by the equation $h(t) = -4.9t^2 + 49t + 2.5$ where the time t is measured in seconds.

- a) What is the height of the rocket after 2 seconds?

$$\begin{aligned} & -4.9(2)^2 + 49(2) + 2.5 \\ & = 80.9 \text{ m} \end{aligned}$$

- b) When does the rocket achieve its maximum height?

$$\begin{aligned} & -4.9(t^2 - 10t + 25 - 25) + 2.5 \\ & = -4.9((t-5)^2 - 25) + 2.5 \\ & = -4.9(t-5)^2 + 122.5 + 2.5 \\ & = -4.9(t-5)^2 + 125 \end{aligned}$$

- c) What is its maximum height?

Max height

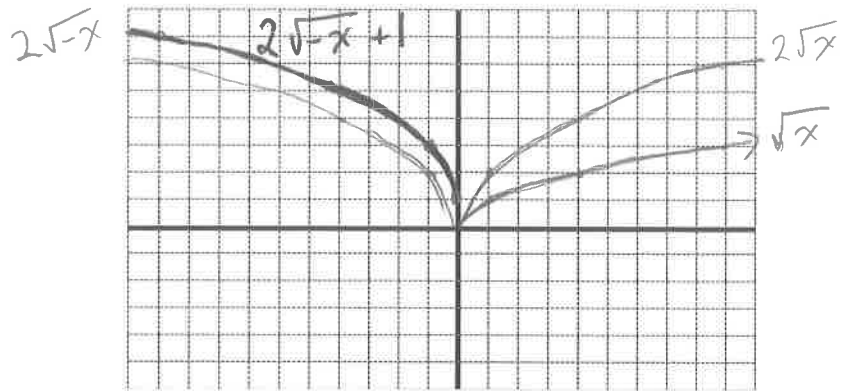
$$= 125 \text{ m}$$

at

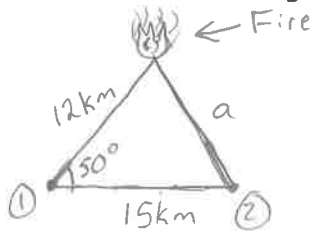
$$5 \text{ seconds}$$

21. By referring to the graph of $y = \sqrt{x}$, sketch the graph of $y = 2\sqrt{-x} + 1$

$$(x, y) \rightarrow \left(\frac{x}{-1}, 2y + 1 \right)$$



22. A forest ranger ^① saw a fire on a bearing of 050° from her position and she estimated that the distance to the fire was approximately 12 km. Another ranger station was due east of her and approximately 15 km from her. How far from the fire was the second ranger station? (Include a diagram.)



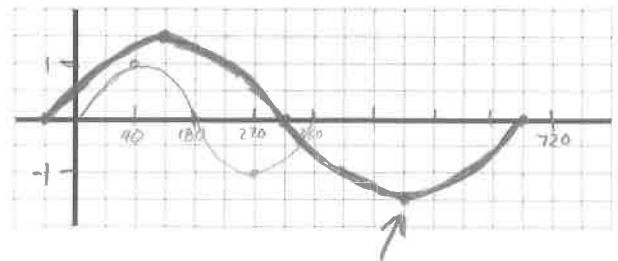
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 12^2 + 15^2 - 2(12)(15) \cos 50^\circ$$

$$a^2 = 137.6$$

$$a = 11.7 \text{ km}$$

23. By referring to the graph of $y = \sin x$, sketch the graph of $y = 1.5 \sin \frac{1}{2}(x + 45)$ showing all critical points and using radian measure on your scale.



$$y = 1.5 \sin \frac{1}{2}(x + 45)$$

24. Solve for x and give exact answers in the indicated measure:

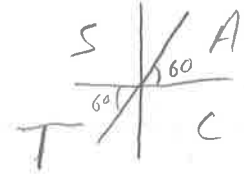
a) $\tan x - \sqrt{3} = 0$, $0^\circ \leq x \leq 360^\circ$

$$\tan x = \sqrt{3}$$

$$x = 60^\circ$$

$$x = 180 + 60$$

$$= 240^\circ$$



b) $2\sin x - \sin x \cos x = 0$, $0 \leq x \leq 360^\circ$

$$\sin x (2 - \cos x) = 0$$

$$\begin{array}{l} \swarrow \\ \sin x = 0 \end{array} \quad \begin{array}{l} \searrow \\ 2 - \cos x = 0 \\ \cos x = 2 \end{array}$$

$$x = 0$$

$$x = 180^\circ$$

$\cos x$ does not
equal 2 for any x

$$\therefore x = 0$$

$$x = 180$$

25. Prove the identity $\frac{2\cos^2 x}{1 - \cos^2 x} = \frac{2}{\tan^2 x}$

26. A car's purchase price is \$24,000. At the end of each year, the value of the car is three-quarters of the value at the beginning of the year.

a) Write the first four terms of the sequence of the car's value at the end of each year.

$$24\,000, 18\,000, 13\,500, 10\,125$$

b) Determine the general term of this sequence to represent the value of the car after n years.

$$t_n = 24000 (0.75)^{n-1}$$

c) Using your general term in b), determine when the car's value will first be less than \$800?

$$800 = 24000 (0.75)^{n-1}$$

$$0.0333 = 0.75^{n-1}$$

trial and error

$n-1 \approx 12$
 $n = 13$

27. Describe how the graph of $y = -2f(3(x-2)) + 2$ can be produced from the graph of $y = f(x)$. List all transformations.

$$y = -2 f (3 (x - 2)) + 2$$

reflection in the x-axis
 Vertical stretch by a factor of 2
 Horizontal compression by 3
 shift Right 2 units
 shift up 2 units

28. Suppose you win a lottery. You have two choices for receiving the money.

Choice 1: \$50 000 at the end of each year for 20 years

Choice 2: \$500,000 now

If current interest rates are approximately 8% p.a. compounded yearly, which is the better choice. Justify your answer with a clear solution and explanation.

$$50000 \times 20$$

$$\text{\$1 000 000}$$

P.V.	-500 000
PMT	50 926.1
F.V.	0
Rate	8
Periods	20
Comp	Annually

If you take Choice #2 you can pay your self more every year.

29. The element Californium "A" has a half life of approximately 45 minutes. How long would it take for 240 mg of Californium to decay to 60 mg?

(Show a full algebraic solution for full marks.)

$$A = A_0 \left(\frac{1}{2}\right)^{t/h}$$

$$60 = 240 \left(\frac{1}{2}\right)^{t/45}$$

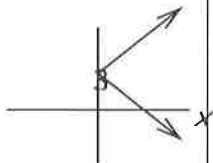
$$\frac{1}{4} = \left(\frac{1}{2}\right)^{t/45}$$

$$\frac{t}{45} = 2$$

$$t = 90$$

$240 \rightsquigarrow 120 \rightsquigarrow 60$
 2 half lifes
 = 90 minutes

30. Complete the following chart

Relation	Function? Yes/No	Domain	Range
a) $\{(4, -5), (6, -5), (0, 7)\}$	Yes	$D \rightarrow \{4, 6, 0\}$	$R \rightarrow \{-5, 7\}$
b) 	No	$D \rightarrow \{x \in \mathbb{R} \mid x \geq 0\}$	$R \rightarrow \{y \in \mathbb{R}\}$
c) $x^2 + y^2 = 49$	No	$x \in \mathbb{R} \mid -7 \leq x \leq 7$	$y \in \mathbb{R} \mid -7 \leq y \leq 7$
d) $y = 4x - 7$	Yes	$x \in \mathbb{R}$	$y \in \mathbb{R}$
e) $y = -\sqrt{x-4}$	Yes	$x \in \mathbb{R} \mid x \geq 4$	$y \in \mathbb{R} \mid y \leq 0$
f) $y = \frac{1}{x}$	Yes	$x \in \mathbb{R} \mid x \neq 0$	$y \in \mathbb{R} \mid y \neq 0$

31. If $f(x) = 5x - 4$ and $g(x) = 3x^2 + 7$, find:

a) $g(-2)$

b) $f(3a)$

c) the value of "x" when $f(x) = -39$

$$g(-2) = 3(-2)^2 + 7$$

$$= 3(4) + 7$$

$$= 12 + 7$$

$$= 19$$

$$f(3a) = 5(3a) - 4$$

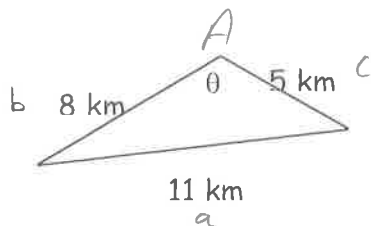
$$= 15a - 4$$

$$-39 = 5x - 4$$

$$-35 = 5x$$

$$-7 = x$$

32. A marathon race follows a triangular course. The 3 legs of the race are 5 km, 11 km and 8 km long. Find the angle θ between the starting and finishing leg.



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{8^2 + 5^2 - 11^2}{2(8)(5)}$$

$$\cos A = \frac{-32}{80}$$

$A = 114^\circ$

33. Find the exact value of each trigonometric ratio.

a) $\tan 150^\circ = -\frac{1}{\sqrt{3}}$

b) $\sin 240^\circ = -\frac{2}{\sqrt{3}}$

c) $\cos 315^\circ = \frac{1}{\sqrt{2}}$



34. If $0^\circ \leq A \leq 360^\circ$, Find the possible measures of $\angle A$.

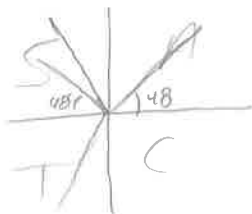
a) $\cos A = -\frac{1}{2}$

b) $\sin A = 0.7459$

$A = 120^\circ$

$A = 180 - 60$

$= 240^\circ$



$A = 48^\circ$

$A = 180 - 48$

$= 132^\circ$

35. Graph $y = 3^x + 2$. Label at least 3 ordered pairs and state the equation of the asymptote.

Asymptote
 $y = 2$

