Characteristics of Polynomial Functions

Graph the following:

$y = x^3$

$y = x^3 + x^2 - 4x - 4$

$y = x^3 + 5x^2 + 3x - 9$

$y = -x^3$

$y = -x^3 - x^2 + 4x + 4$

$y = -x^3 - 5x^2 - 3x + 9$

Note similarities and difference of
- End behaviour
- Number of local extrema (max or min)
- Number of x intercepts

Characteristics of Polynomial Functions

Graph the following:

$y = x^4$

$y = x^4 - x^3 - 6x^2 + 4x + 8$

$y = x^4 - 3x^3 - 3x^2 + 11x - 4$

$y = -x^4$

$y = -x^4 + x^3 + 6x^2 - 4x - 8$

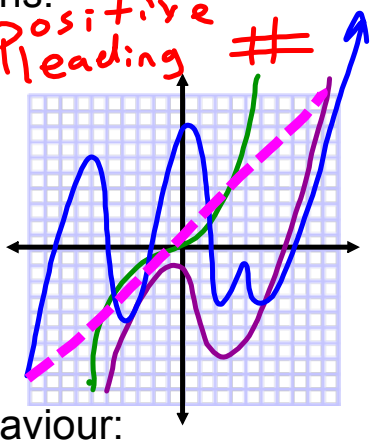
$y = -x^4 + 3x^3 + 3x^2 - 11x + 4$

Note similarities and difference of
- End behaviour
- Number of local extrema (max or min)
- Number of x intercepts

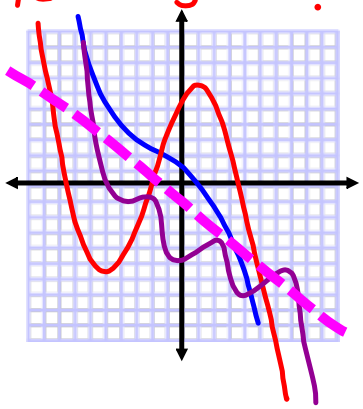
Odd Degree Functions

Graphs:

positive leading #



negative leading #.



End Behaviour:

Q3 → Q1 | Q2 → Q4

Number of x-intercepts:

between 1 and n (degree n)

Domain and Range:

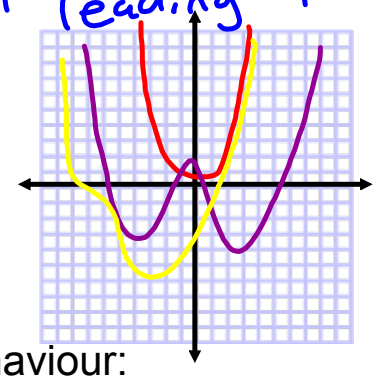
D → $(-\infty, \infty)$ R → $(-\infty, \infty)$

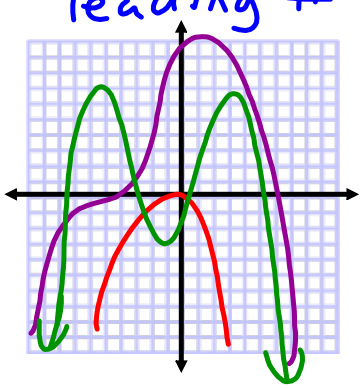
Extrema:

between 0 and n - 1

Even Degree Functions

Graphs:





End Behaviour:

Q2 → Q1

Q3 → Q4

Number of x-intercepts:

between 0 and n

Domain and Range:

D → $\{-\infty, \infty\}$

Range → $\{a, \infty\}$ where "a" is the global max or min

→ $\{-\infty, a\}$

Extrema:

between 1 and n-1

Finite Differences

Construct a table of values and determine the finite differences for the function

$f(x) = 2x^3 - 4x^2 + x + 1$

x	y	First Diff	Second Diff	Third Diff
0	1			
1	0	-1	4	12
2	3	3	16	12
3	22	19	28	12
4	69	47	40	
5	156	87		

Degree 3

If the n^{th} differences are constant, the function is degree n.

Third Diff = 12

Degree = 3

leading # = 2

$12 = a(n!)$

$2(3!)$

$= 2(3 \times 2 \times 1)$

$= 12$

For any polynomial function of degree n, the nth differences

- are constant
- have the same sign as the leading coefficient
- are equal to a(n!), where a is the leading coefficient , n → degree

$$n! = n(n-1)(n-2)(n-3)\ldots(2)(1)$$

For the following table of values determine
1. the degree of the polynomial function
2. the value (and sign) of the leading coefficient

4th

x	y
-2	-54
-1	-8
0	0
1	6
2	22
3	36
4	12

F.D S.D T.D Fourth D.ff

46 > -38 > 36 > 24
8 > -2 > 12 > 24
6 > 10 > -12 > 24
16 > -2 > -36
14 > -38
-24

$-24 = a(n!)$

$-24 = a(4!)$ → $4 \times 3 \times 2 \times 1 = 24$

$-24 = a(24)$

$-1 = a$

Use the regression feature of Desmos to determine an equation for the table

$y = ax + b$

$y = ax^2 + bx + c$

$y = ax^3 + bx^2 + cx + d$

$y = ax^4 + bx^3 + cx^2 + dx + e$

Homework

pg. 26 # 1, 3, 7, 11, 12, 15