MCR 3U **Applications of Exponential Functions** Day 4

There are many things in the real-world that model exponential functions…

Examples:

- Population growth/decline (i.e., the population of Port Colborne) 2%/yr
- Finances (i.e., compound interest) 5% interest
- Half-life Carbon atom (600 years)

We can use exponential equations to model exponential **growth** or **decay,** using the following formula:

$$y = ab^{\frac{x}{t}} \qquad f(t) = ab^{\frac{t}{h}}$$

Where:

a is the initial value of the scenario.

b is the growth or decay ***rate.***

x is an independent variable.

t is the growth period.

How do we know if a situation is exponential growth or decay?

- If $b > 1$, then the function is experiencing exponential ***GROWTH.***
- If $0 < b < 1$, then the function is experiencing exponential ***DECAY.***

We can use exponential functions to solve problems arising from real-world contexts:

Example 1:

The cost of a new Mercedes-Benz is modelled by the function $C(t) = 70000(0.81)^{t}$, where t represents time in years, and $C(t)$ represents the cost of the Mercedes-Benz, in dollars.

a) What is the initial cost of the Mercedes-Benz?

\$70000 $C(0) = 70000(0.81)^{0} = \boxed{70000}$

b) Is this an example of exponential growth or decay?

decay

c) What is the growth/decay rate? (i.e., what percentage is the cost of the Mercedes-Benz increasing or decreasing?)

19% $(1 - 0.81) = 0.19$

$C(t) = 70000(0.81)^t$

d) What is the cost of the Mercedes-Benz after 3 years?

$C(3) = 70000(0.81)^3$

$= \boxed{\$37{,}200.87}$

e) How many years does it take for the cost of the Mercedes-Benz to depreciate to \$8510? Round your answer to the nearest year.

$\frac{8510}{70000} = \frac{70000(0.81)^t}{70000}$ $(0.81)^t = 0.1216$

$t = 6 \Rightarrow 0.28$

$t = 8 \Rightarrow 0.185$

$t = 10 \Rightarrow 0.1215$

$\boxed{t = 10}$

Example 2:

b a h

The half-life of mercury is 50 years (this means it takes 50 years for mercury to reduce to half of its mass). If there is 50 grams of the element initially, how much will be left in 1000 years?

t

$m(t) = 50\left(\frac{1}{2}\right)^{\frac{t}{50}}$

$m(1000) = 50\left(\frac{1}{2}\right)^{\frac{1000}{50}}$

$\boxed{m(1000) = 0.0000476837\,g}$

Example 3:

The population of Port Colborne is expected to double every 30 years. If the initial population of Port Colborne is 18,500 in 2019, what is the expected population of Port Colborne in 2090?

b

$P(t) = 18500(2)^{\frac{t}{30}}$ $t = 71$

$P(71) = 18500(2)^{\frac{71}{30}}$

$= \boxed{93733}$

Example 4:

Olivia puts $1000 in a savings account and collects 8% interest every 2 years.

a) Write an equation that best represents this situation.

$S(t) = 1000(1.08)^{t/2}$

b) How much does Olivia have in her savings account after 10 years?

$S(10) = 1000(1.08)^{\frac{10}{2}}$

$= \boxed{\$1469.33}$

c) How long does it take for Olivia's savings account to get to $43,427? Round to the nearest year.

$$\frac{43427}{1000} = \frac{1000(1.08)^{\frac{t}{2}}}{1000}$$

y = left side

y = right side

P
E
S
M
O
S

$$\boxed{43.427 = (1.08)^{\frac{t}{2}}}$$

$t \doteq 98$ years

$$\log_{10}(43.427) = \log_{10}(1.08)^{\frac{t}{2}}$$

$$2\log(43.427) = \frac{t}{\cancel{2}}\log(1.08)$$

$$t = \frac{2\log(43.427)}{\log(1.08)}$$

$$\boxed{t \doteq 98}$$