Why might Remainder Theorem be Important?

if $P(x)$ is divided by $x-z$
the remainder is $P(z)$

Factor Theorem

(x - b) is a factor of P(x) iff P(b) = 0 if and only if

Similarly, (ax - b) is a factor of P(x) iff P($\frac{b}{a}$) = 0

Factor $f(x) = x^3 + 3x^2 + 4x - 8$

$f(b) = 0$

$f(1) =$

$f(2) =$

$f(-1)$

$f(-2)$

$(x+1)$

Rational Zero Theorem

Suppose P(x) is a polynomial function with integer coefficients and $\frac{a}{b}$ is a zero of P(x), where a and b are both integers. Then,

* b is a factor of the constant term of P(x)
* a is a factor of the leading coefficient of P(x)
* ax - b is a factor of P(x)

Factor the following

$$x^3 + 2x^2 - x - 2$$

$z = -2$ $\quad a = 1$

± 1 $\quad 1$

± 2

OPTIONS FOR FACTORS

P(1) P(2)

P(-1) P(-2)

$P(1) = (1)^3 + 2(1)^2 - (1) - 2$

$= 0$

$\therefore (x-1)$ is a factor

$P(-1) = (-1)^3 + 2(-1)^2 - (-1) - 2$

$= 0$

$\therefore (x+1)$ is a factor

$P(-2) = (-2)^3 + 2(-2)^2 - (-2) - 2$

$= -8 + 8 + 2 - 2$

$= 0$

$\therefore (x+2)$ is a factor

So

$x^3 + 2x^2 - x - 2 = (x+1)(x-1)(x+2)$

$4x^3 + 3x^2 - 4x - 3$

-3 $\quad$ 4

$P(1) = 4(1)^3 + 3(1)^2 - 4(1) - 3$

$= 0$

$\therefore (x-1)$ is a factor

$z = \{\pm 1, \pm 3\}$ $\quad$ $a = \{\pm 1, \pm 2, \pm 4\}$

Check

$P(1)$	$P(\frac{1}{2})$	$P(\frac{1}{4})$
$P(-1)$	$P(-\frac{1}{2})$	$P(-\frac{1}{4})$
$P(3)$	$P(\frac{3}{2})$	$P(\frac{3}{4})$
$P(-3)$	$P(-\frac{3}{2})$	$P(-\frac{3}{4})$

$$\begin{array}{r|l} & 4x^2 + 7x + 3 \\ \hline x-1 & 4x^3 + 3x^2 - 4x - 3 \\ & -4x^3 - 4x^2 \\ \hline & 0 \quad 7x^2 - 4x \\ & \quad -7x^2 - 7x \\ \hline & \quad 0 \quad 3x - 3 \\ & \quad\quad -3x - 3 \\ \hline & \quad\quad 0 \end{array}$$

$4x^3 + 3x^2 - 4x - 3$

$= (x-1)(4x^2 + 7x + 3)$

$= (x-1)(x+1)(4x+3)$

$4x^2 + 7x + 3$ ($3 \times 4 = 12$)

⊗ 12
⊕ 7
3, 4

$= 4x^2 + 4x + 3x + 3$

$= 4x(x+1) + 3(x+1)$

$= (x+1)(4x+3)$

Homework

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