MCR 3U **Rational Exponents**

Minds On!

Consider this…

How can we write 4^1 as a product of two powers with base 4?

$4^1 = 4^? 4^?$ … What is ?. $a^m a^n = a^{m+n}$

$? = \frac{1}{2}$ $4^{\frac{1}{2}} 4^{\frac{1}{2}} = 4^1$ $\sqrt{4} = 2$

Therefore, the rational exponent $\frac{1}{2}$ is the same as a Square root.

$8^1 = 8^? 8^? 8^?$

$? = \frac{1}{3}$ $\sqrt[3]{8} = 2$

Therefore, the rational exponent $\frac{1}{3}$ is the same as a Cube root.

$81^1 = 81^? 81^? 81^? 81^?$

$? = \frac{1}{4}$ $\sqrt[4]{81} = 3$

Therefore, the rational exponent $\frac{1}{4}$ is the same as a fourth root.

Do we notice a pattern?

$x^{\frac{1}{n}} = \sqrt[n]{x}$

$x^{\frac{m}{n}} = (x^{\frac{1}{n}})^m$

$= (\sqrt[n]{x})^m$ OR $\sqrt[n]{x^m}$

(*)

In general, when working with rational exponents:

$x^{\frac{1}{n}} = \sqrt[n]{x}$

$x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$ OR $x^{\frac{m}{n}} = \sqrt[n]{x^m}$

Action!

Example 1:

Write the following in radical form.

a) $x^{\frac{3}{5}} = \left(\sqrt[5]{x}\right)^3$ b) $-a^{\frac{1}{2}} = -\sqrt{a}$

Example 2:

Write the following in exponential form.

a) $\sqrt{z} = z^{\frac{1}{2}}$ b) $\left(\sqrt[4]{a}\right)^5 = a^{\frac{5}{4}}$

In English, if we have $\sqrt[3]{81}$, this means "what are two of the same numbers that multiply that gi
81?

So what about $\sqrt[3]{8}$?

what are three of the same # that multiply to give 8?

$\sqrt[3]{8} = 2$ (b/c $2 \times 2 \times 2 = 8$)

Example 3:

Evaluate without the use of a calculator.

$3^{-2} = \frac{1}{3^2}$

a) $25^{\frac{1}{2}}$ b) $8^{\frac{2}{3}}$ c) $81^{-\frac{1}{4}}$ d) $(-27)^{\frac{-2}{3}}$ e) $\left(\frac{4}{9}\right)^{-\frac{1}{2}}$ f) $\left(\frac{16}{81}\right)^{-\frac{3}{4}}$

a) $\sqrt{25} = 5$

b) $(\sqrt[3]{8})^2 = 2^2 = 4$

c) $\left(\frac{1}{81}\right)^{\frac{1}{4}} = 4\sqrt{\frac{1}{81}} = \frac{1}{3}$

d) $\left(\frac{-1}{27}\right)^{\frac{2}{3}} = \left(\sqrt[3]{\frac{-1}{27}}\right)^2 = \left(\frac{-1}{3}\right)^2 = \frac{1}{9}$

Remember! Exponent laws still apply. Also, it would be helpful to memorize square/cube root

Homework: Section 3.3, Page 175-176, #1-6

e) $\left(\frac{4}{9}\right)^{-\frac{1}{2}}$ f) $\left(\frac{16}{81}\right)^{-\frac{3}{4}}$

e) $= \left(\frac{9}{4}\right)^{\frac{1}{2}}$

$= \sqrt{\frac{9}{4}} = \boxed{\frac{3}{2}}$

f) $\left(\frac{81}{16}\right)^{\frac{3}{4}}$

$= \left(\sqrt[4]{\frac{81}{16}}\right)^3 = \left(\frac{3}{2}\right)^3 = \boxed{\frac{27}{8}}$