Special Triangles/Angles

MCR 3U

Minds On!

$$\frac{2}{\sqrt{3}}$$

- By yourself (on in pairs) solve all for all angles and side lengths in the following triangles. Make sure your answers are _**EXACT**_ (i.e., no decimals).
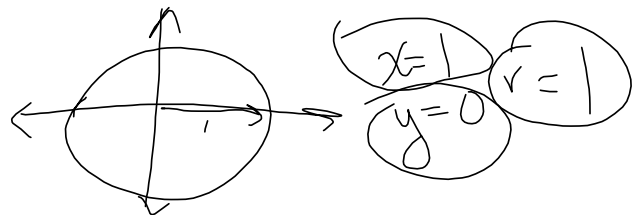
45° C $\sqrt{2}$

1

a

45°

60°

1

2

30°

$\sqrt{3}$
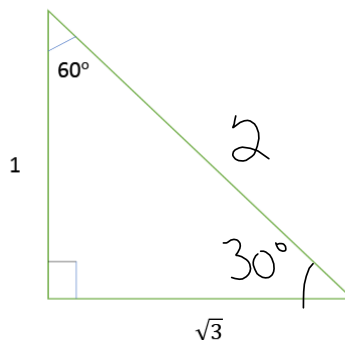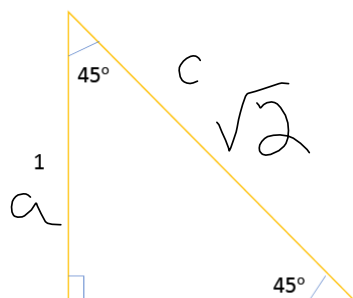
$$a^2 + b^2 = c^2$$
$$1^2 + 1^2 = c^2$$
$$\sqrt{2} = \sqrt{c^2}$$
$$c = \sqrt{2}$$

1

1

Continued...

• Complete the table:

	0°	30°	45°	60°	90°
sin	0	$\dfrac{1}{2}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{\sqrt{3}}{2}$	1
cos	1	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{1}{2}$	0
tan	0	$\dfrac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef.

What happens when the angle equals 0 degrees? What is sin(0)? What is cos(0)? Illustrate with a unit circle.

$x=1$
$y=0$ $r=1$

What happens when the angle equals 90 degrees? What is sin(90)? What is cos(90)? Illustrate with a unit circle.

$x=0$
$y=1$ $r=1$
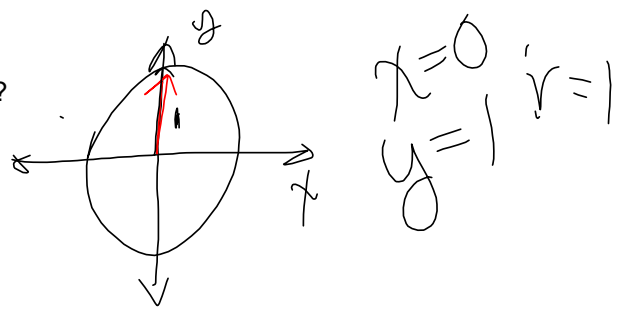
$$x^2 + y^2 = r^2$$
$$1^2 + 0^2 = r^2$$
$$r^2 = 1$$
$$\boxed{r = 1}$$

What do we notice?

- a) Find a relationship between the special angles of sine and cosine.

$$\text{Values are reversed for } 0-90°$$

- b) Apply an operation to the special angles of sine and cosine (i.e., add, subtract, multiply or divide them) and compare them with the special angles of the tangent ratio. What do you notice?

$$? \quad \sin(30) + \cos(30) = \tan(30)?$$

$$\frac{\sin(30)}{\cos(30)} \overset{?}{=} \tan(30)$$

$$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\frac{\frac{y}{r}}{\frac{x}{r}}$$

$$\frac{y}{r} \cdot \frac{r}{x} = \frac{y}{x}$$

Examples

- Simplify to determine the EXACT value of the following expressions. Show all steps as if you were not using a calculator.

a) $(sin30°)(cos45°)(tan60°)$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{1}\right)$$

$$= \boxed{\frac{\sqrt{3}}{2\sqrt{2}}}$$

$$2^1 \; 2^{-\frac{1}{2}}$$

$$= 2^{\frac{3}{2}}$$

$$\frac{1}{\sqrt{2}}\left(\frac{\times\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{2}$$

b) $\sin(135°) + \cos(120°) + \tan(180°)$

$$= \frac{1}{\sqrt{2}} + \left(\frac{-1}{2}\right) + 0$$

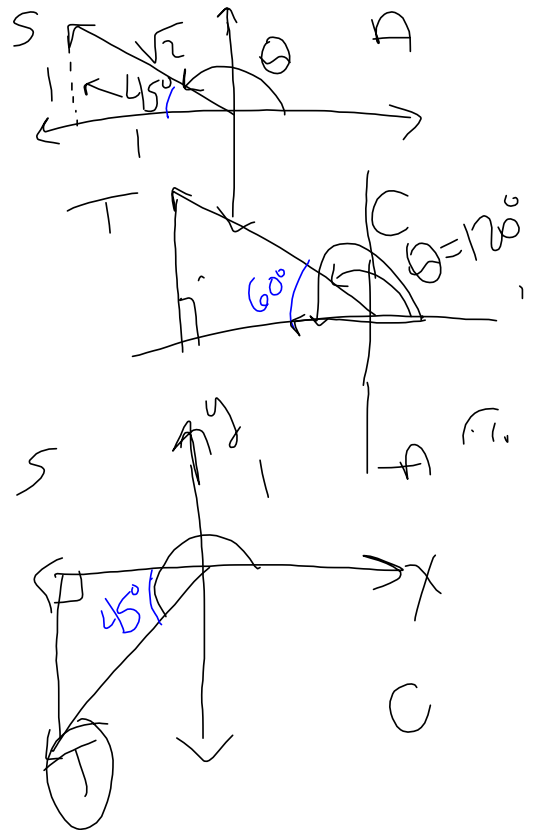$$= \frac{2}{2\sqrt{2}} - \frac{\sqrt{2}}{2\sqrt{2}} = \boxed{\frac{2-\sqrt{2}}{2\sqrt{2}}}$$

c) $\csc(60°) + \sec(45°)\csc(225°)$

$$= \frac{1}{\sin 60°} + \frac{1}{\cos 45°} \cdot \frac{1}{\sin 225°}$$

$$= \frac{2}{\sqrt{3}} + \left(\frac{\sqrt{2}}{1}\right)\left(\frac{-\sqrt{2}}{1}\right)$$
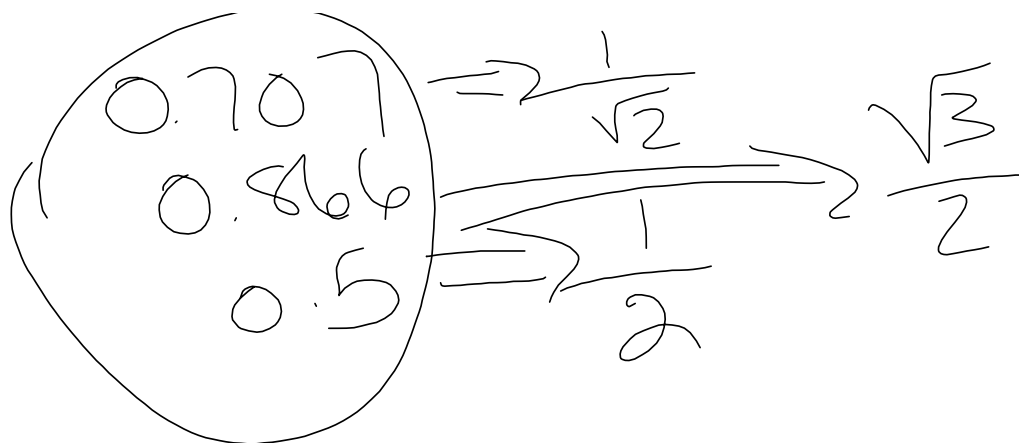
$$= \frac{2}{\sqrt{3}} - \frac{2}{1}$$

$$= \frac{2}{\sqrt{3}} - \frac{2\sqrt{3}}{\sqrt{3}} = \boxed{\frac{2-2\sqrt{3}}{\sqrt{3}}}$$

Homework

- Complete Worksheet

$$0.707 \Rightarrow \frac{1}{\sqrt{2}}$$

$$0.866 \Rightarrow \frac{\sqrt{3}}{2}$$

$$0.5 \Rightarrow \frac{1}{2}$$

$$\frac{\cancel{x}(3)}{\cancel{x}} \qquad \frac{\cancel{x}-3}{\cancel{x}}$$