Trigonometric Identities

MCR 3U

What is an identity?

- An identity is a mathematical statement that is true for all variables in the expression that is within its domain.

LS=RS
for all values

Minds On!

- Using the unit circle formulas for the trigonometric ratios (i.e., $x, y \ and \ r$) how do we express the trigonometric ratios in terms of $x, y \ and \ r$?

$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$

Minds On! Con't…

- Using the formulas that you have made, prove the following…

$(\sin\theta)^2$

a) $\sin^2\theta + \cos^2\theta = 1$

LS

$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$

$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$

$= \frac{x^2+y^2}{r^2}$ $= r^2$

$= \frac{r^2}{r^2} = 1$

RS

$= 1$

$\therefore$ LS=RS

$\because \rightarrow$ since

$\therefore \rightarrow$ therefore

b) $1 + \tan^2\theta = \sec^2\theta$

LS

$= 1 + \left(\frac{y}{x}\right)^2$

$= 1 + \frac{y^2}{x^2}$

$= \frac{x^2}{x^2} + \frac{y^2}{x^2}$

$= \frac{x^2 + y^2}{x^2}$

$= \frac{r^2}{x^2}$ ✓

RS

$= \left(\frac{r}{x}\right)^2$

$= \frac{r^2}{x^2}$ ✓

c) $tan\theta = \frac{sin\theta}{cos\theta}$

LS

$= \frac{y}{x}$

RS

$\frac{\frac{y}{r}}{\frac{x}{r}}$

$= \frac{y}{\cancel{r}} \cdot \frac{\cancel{r}}{x}$

$= \frac{y}{x}$

Things to consider… Important!

- There is no direct way to solve trigonometric identities! So what can we do?
  - 1. Change ALL reciprocal ratios into primary ratios immediately.
  - 2. Use $tan\theta = \frac{sin\theta}{cos\theta}$ to simplify. If you use step 1 and 2, you will have expressions that are only in terms of sin and cos (2 trig ratios) as opposed to working with 6 ratios!
  - 3. Factor if possible!
  - 4. Find a common denominator (if applicable).

Formulas

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $\tan\theta = \frac{\sin\theta}{\cos\theta}$

1. Prove the following trig identity:

$$\cos^2\theta \sec\theta = \cot\theta \sin\theta$$

LS

$= \cos^{\cancel{2}}\theta\left(\frac{1}{\cancel{\cos\theta}}\right)$

$= \cos\theta$ ✓

RS

$= \left(\frac{1}{\tan\theta}\right)\sin\theta$

$= \left(\frac{1}{\frac{\sin\theta}{\cos\theta}}\right)\sin\theta$

$= \left(\frac{\cos\theta}{\cancel{\sin\theta}}\right)\cancel{\sin\theta}$

$= \cos\theta$ ✓

$\therefore LS = RS$

2. Prove the following trig identity:

$$tan\beta = \frac{sin\beta + \sin^2\beta}{cos\beta(1 + sin\beta)}$$

GCF = $\sin\beta$ $x + x^2$

$x(1+x)$

LS

$= \frac{\sin\beta}{\cos\beta}$ ✓

RS

$= \frac{\sin\beta\cancel{(1+\sin\beta)}}{\cos\beta\cancel{(1+\sin\beta)}}$

$= \frac{\sin\beta}{\cos\beta}$ ✓

$\therefore LS = RS$

3. Prove the following trig identity:

$$sin\theta - cos\theta = \frac{\sin^4\theta - \cos^4\theta}{sin\theta + cos\theta}$$

LS

$= \sin\theta - \cos\theta$

$\therefore LS = RS$

RS

$a^2 - b^2 = (a+b)(a-b)$

$= \frac{(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)}{\sin\theta + \cos\theta}$

$\sqrt{\sin^4\theta} = \sin^2\theta$

$\sqrt{\cos^4\theta} = \cos^2\theta$

$= \frac{(1)(\sin^2\theta - \cos^2\theta)}{\sin\theta + \cos\theta}$

$= \frac{\cancel{(\sin\theta + \cos\theta)}(\sin\theta - \cos\theta)}{\cancel{\sin\theta + \cos\theta}}$

$= \sin\theta - \cos\theta$

4. Prove the following trig identity:

$$\frac{\cos^2 x}{sinx} + \frac{sin^2 x}{sinx} = cscx$$

LS

$= \frac{\cos^2 x + \sin^2 x}{\sin x}$

$= \frac{1}{\sin x}$ ✓

RS

$= \frac{1}{\sin x}$ ✓

5. Prove the following trig identity:

$$\sin^2\alpha = \frac{\tan^2\alpha}{1+\tan^2\alpha}$$

$1+\tan^2\alpha = \sec^2\alpha$

LS

$=\sin^2\alpha$

$\therefore LS = RS$

RS

$$= \frac{\frac{\sin^2\alpha}{\cos^2\alpha}}{\sec^2\alpha}$$

$$= \frac{\frac{\sin^2\alpha}{\cos^2\alpha}}{\frac{1}{\cos^2\alpha}}$$

$$= \frac{\sin^2\alpha}{\cancel{\cos^2\alpha}} \cdot \frac{\cancel{\cos^2\alpha}}{1}$$

$= \boxed{\sin^2\alpha}$

NOTE

- You may have to switch around some identities (such as the Pythagorean identity) to prove some trig identities.

$\mathbf{\sin^2 x + \cos^2 x = 1}$ can also be modified as $\mathbf{\sin^2 x = 1 - \cos^2 x}$ or $\mathbf{\cos^2 x = 1 - \sin^2 x}$.

LS

$\cos\theta(1-\sin^2\theta)$

$=\cos\theta(\cos^2\theta)$

Homework

- Section 4.6, Page 273-274, #3-9

Note

$\sin^2\theta+\cos^2\theta=1$

$\sin^2\theta=1-\cos^2\theta$

$\cos^2\theta=1-\sin^2\theta$