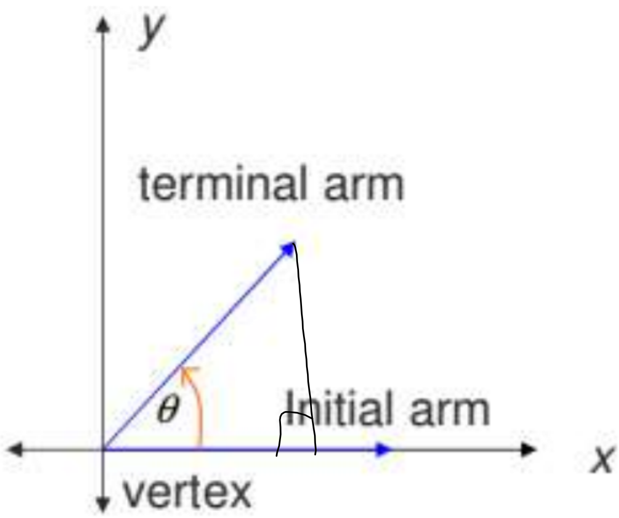
The Unit Circle: Angles Between 0° and 360°

MCR 3U

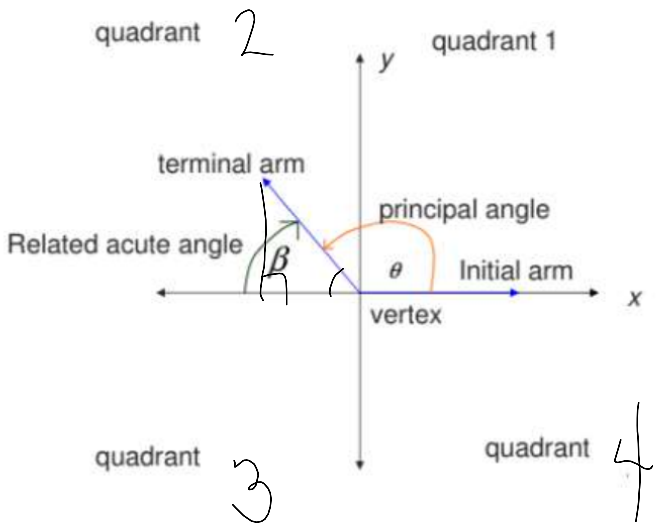
Definitions

- Initial Arm: The arm that stays STATIONARY (i.e., does not move) which lies on the positive x-axis.
- Terminal Arm: The arm that rotates depending on the measure of angle θ.



Definitions con't

- Principal Angle: The angle between the terminal arm and the positive x-axis (measured counter-clockwise)
- Related Acute Angle: The angle between the terminal arm of an angle and the x-axis, if the principal angle is greater than $90°$.



Recall

- What is the equation of a circle?

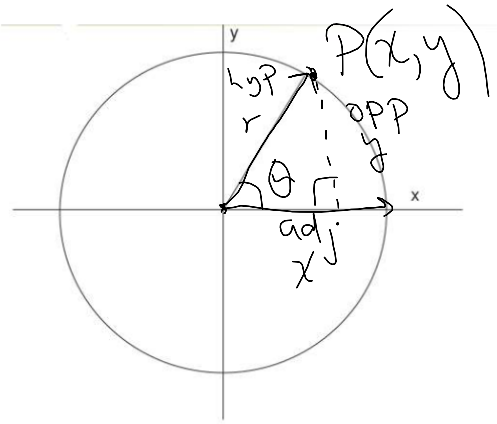
$x^2 + y^2 = r^2$

- Does this look similar to another formula we know?

$a^2 + b^2 = c^2$

Introduction to the Unit Circle

- Assume we take a circle with radius 1. In general, how can we write the expressions for $\sin(\theta)$, $\cos(\theta)$ and $\tan(\theta)$? What about the reciprocal trig ratios?



$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$

$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$

$\tan\theta = \frac{opp}{adj} = \frac{y}{x}$

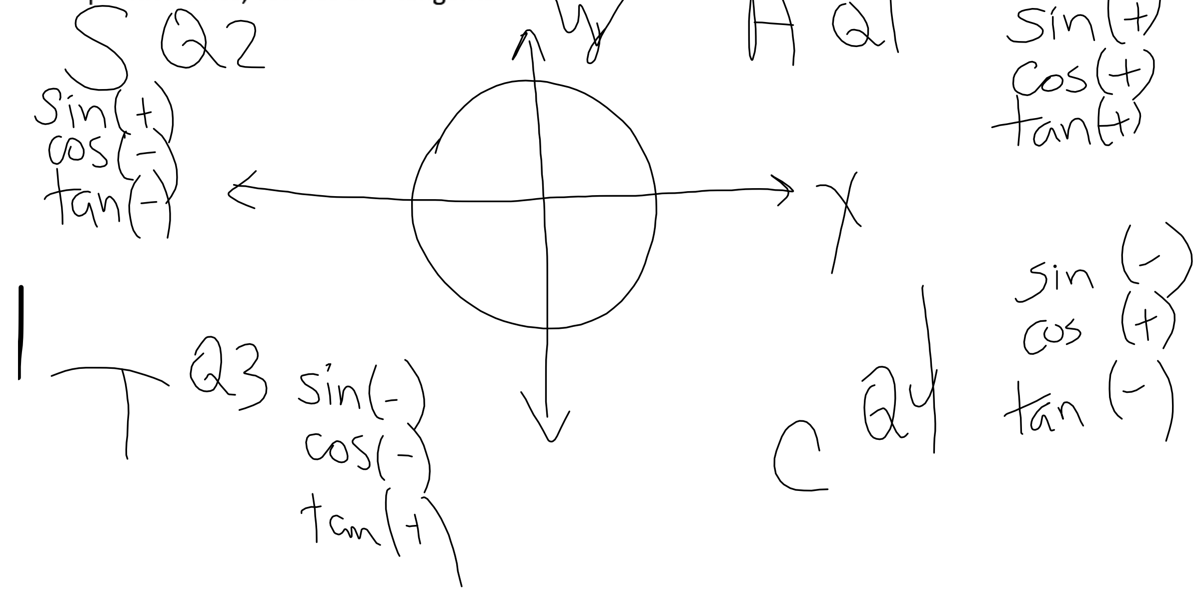
$\csc\theta = \frac{r}{y}$

$\sec\theta = \frac{r}{x}$

$\cot\theta = \frac{x}{y}$

The CAST Rule

- The calculator always wants to give us <u>ACUTE</u> angles when we look them up. But, in reality, it is possible to have <u>OBTUSE</u> angles as well. We call these angles **co-terminal angles** to each other.
- This leads us to what we call the CAST rule. It tells us in which quadrant angles will have positive sines, cosines and tangents.



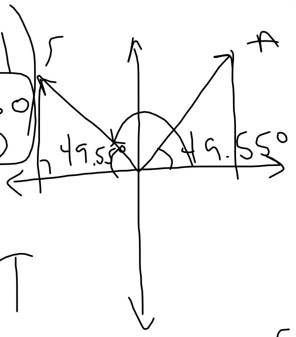
Example

Solve for θ, $0° \leq \theta \leq 360°$.

a) $\sin(\theta) = 0.761$

$\theta = \sin^{-1}(0.761)$

$\theta = \boxed{49.55°}$



$\theta = 180° - 49.55°$ T

$= \boxed{130.45°}$

b) $\sec(\theta) = -3$ → $\frac{-3}{1}$

$\cos\theta = \frac{-1}{3}$

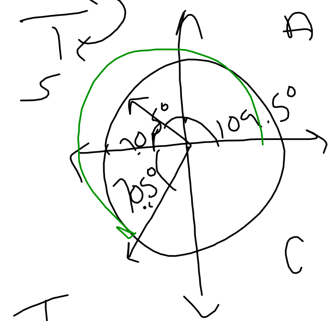
$\theta = \cos^{-1}\left(\frac{-1}{3}\right)$

C

$\boxed{\theta = 109.5°}$

$\theta = 180° + 70.5°$

$\boxed{\theta = 250.5°}$

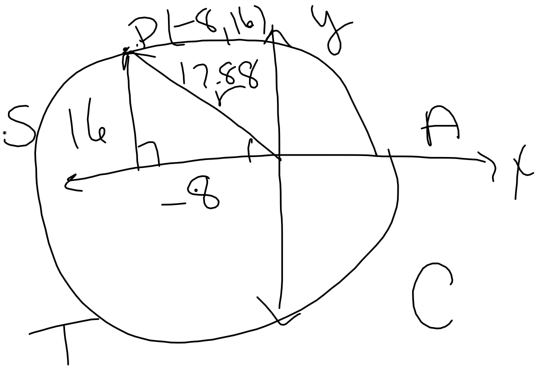


Example 2

- P(-8,16) lies on the terminal arm of angle θ.

a) Determine the ***EXACT*** values for all six trigonometric ratios for θ.

b) Solve for all possible values of θ, $0° \leq \theta \leq 360°$.



a) $x^2 + y^2 = r^2$

$(-8)^2 + (16)^2 = r^2$

$64 + 256 = r^2$

$r^2 = 320$

$r = \sqrt{320}$

$r = 17.88$

$\sin\theta = \frac{y}{r} = \frac{16}{17.88} = 0.89$ $\quad \csc\theta = \frac{17.88}{16} = 1.12$

$\cos\theta = \frac{x}{r} = \frac{-8}{17.88} = -0.45$ $\quad \sec\theta = \frac{17.88}{-8} = -2.22$

$\tan\theta = \frac{y}{x} = \frac{16}{-8} = -2$ $\quad \cot\theta = \frac{-8}{16} = -0.5$

b) $\tan\theta = -2$

$\theta = \tan^{-1}(-2)$

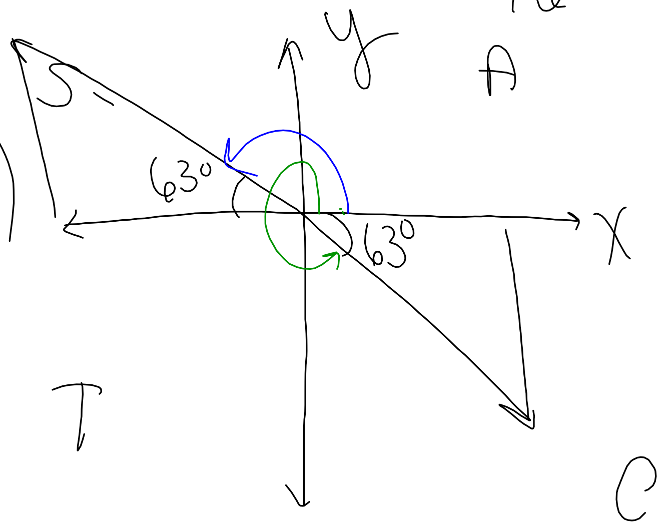
$\theta = -63°$

$\theta = 360° - 63°$

$= 297°$

$\theta = 180° - 63°$

$= 117°$



Example 3

- If $\csc(\theta) = \frac{13}{12}$, what is the value of $\cos(\theta)$? Illustrate with a diagram.

$\sin\theta = \frac{12}{13}\ \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$x^2 + y^2 = r^2$

$x^2 + 12^2 = 13^2$

$x^2 + 144 = 169$

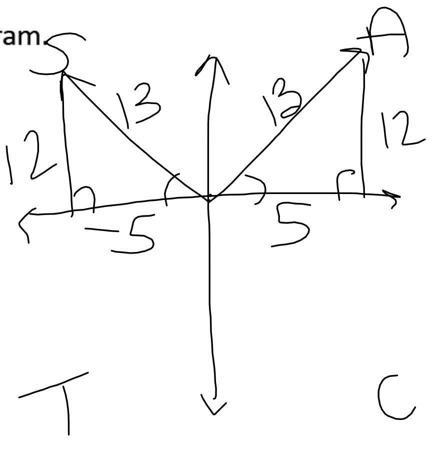
$x^2 = 25$

$x = \pm 5$

$\cos\theta = \frac{5}{13}$

OR

$\cos\theta = \frac{-5}{13}$



Homework

- Section 4.2, Page 237-239, #1-5
- NOTE: For #1-3, write down ALL SIX TRIGONOMETRIC RATIOS, not just the three primary trigonometric ratios!!!